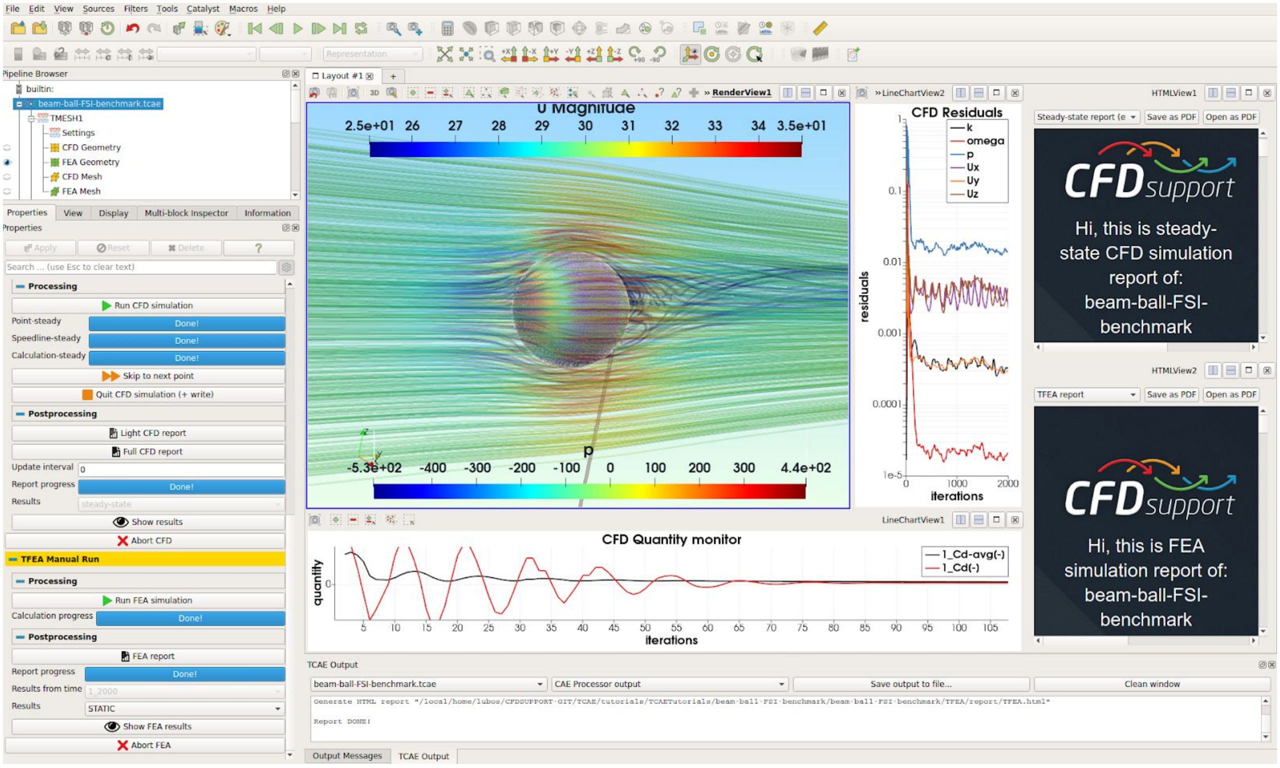
Beam-Ball FSI Benchmark

This report presents the FSI (Fluid-Structure Interaction) validation benchmark, using TCAE simulation software. This study is unique, because the exact solution can be evaluated analytically, and the analytical solution can be compared with the simulation results. The particular goal of this study is to investigate and compare the deformation of a beam with a ball at its end, stressed by the airflow.



Hi, this is steady-state CFD support report of: beam-ball-FSI-benchmark

Hi, this is FEA simulation report of: beam-ball-FSI-benchmark

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Generate HTML report "/local/home/lubos/CFDSUPPORT-GIT/TCAE/tutorials/TCAETutorials/beam-ball-FSI-benchmark/beam-ball-FSI-benchmark/TFEA/report/TFEA.html"

Report DONE!
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Keywords

CFD, FEA, FSI, VALIDATION, BENCHMARK, TCAE, TMESH, TCFD, TFEA, SIMULATION, BEAM, BALL, SPHERE, DEFORMATION, DISPLACEMENT, STRESS, MODAL ANALYSIS, INCOMPRESSIBLE, RANS, AIRFLOW, STEADY-STATE, AUTOMATION, WORKFLOW

Benchmark Parameters

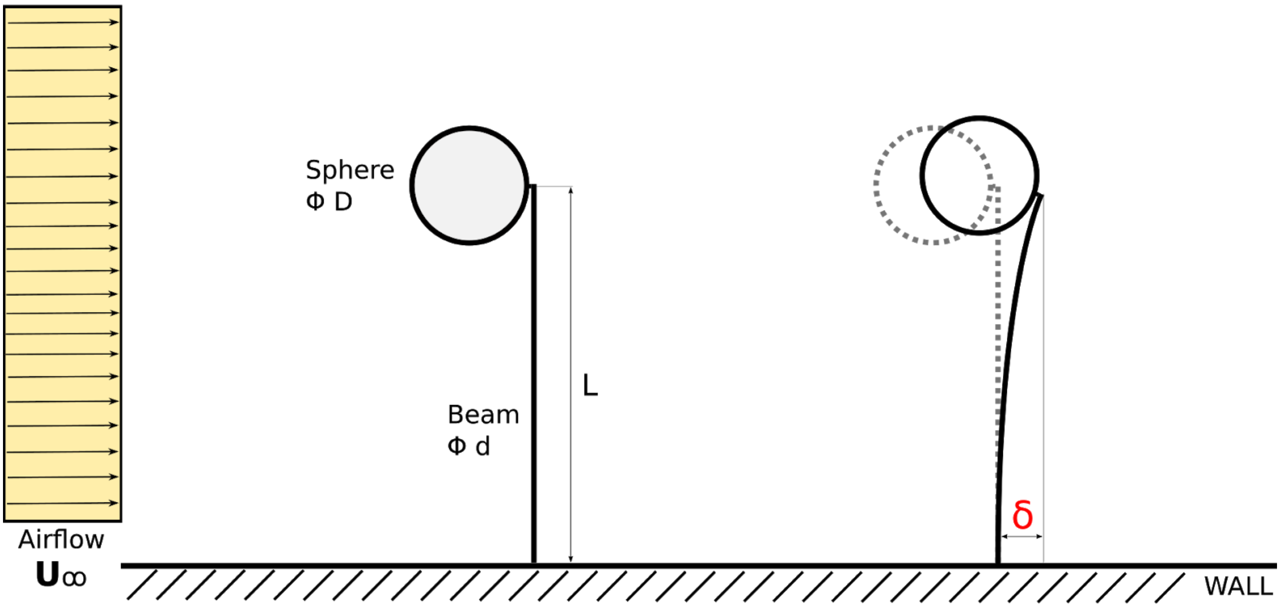
- Airflow speed: *30 m/s*
- Flow model: in*compressible*
- CFD Mesh size: *0.5M cells*
- Medium: *air*
- Dynamic viscosity: *1.8 × 10*$^{-5}$ *Pa · s*
- Air density: *1.2 kg/m*^{3}
- Turbulence intensity: 1%
- Turb. Model: *k-omega SST*
- Beam material: *steel*
- Material density: *7800 kg/m*^{3}
- Material structure: *isotropic*
- Young modulus: *2.1E11 Pa*
- Poisson ratio: *0.3*
- Simulation type: *Virtual Tunnel*
- FEA Mesh size: *0.2M cells*
- Total CPU Time: *2 core.hours*

Beam-Ball Benchmark Introduction

This study is unique, because the exact solution can be evaluated analytically, and the analytical solution can be compared with the simulation results. The particular goal of this benchmark is to investigate and compare the deformation of a beam with a ball at its end, stressed by the airflow.

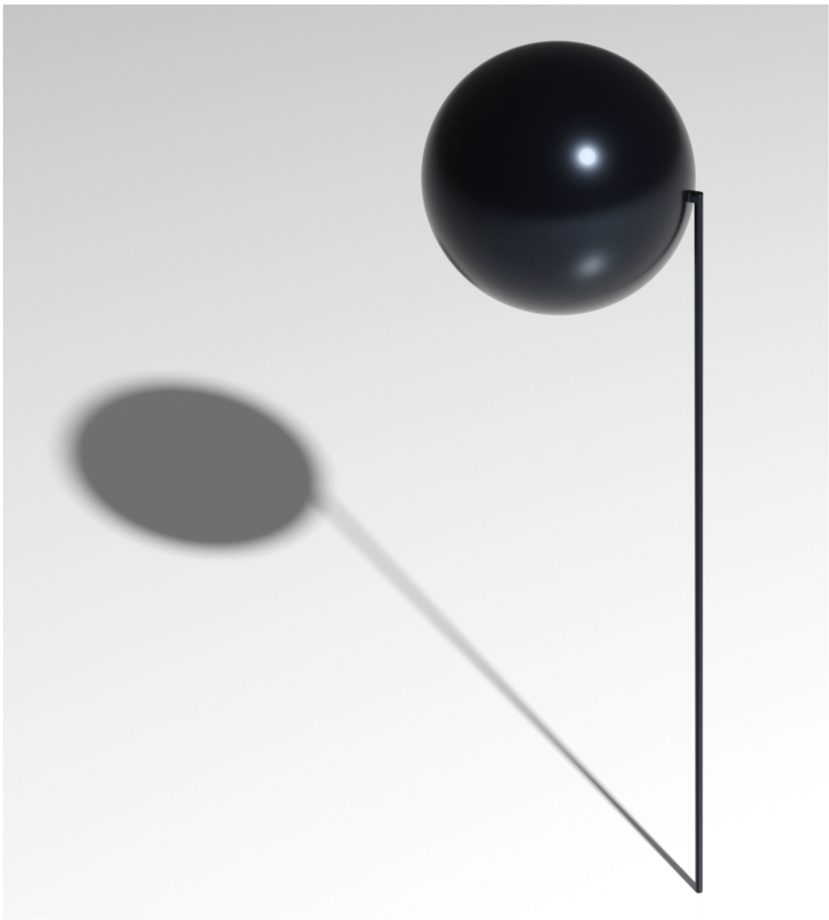
Let’s consider the following case as shown in the simple sketch below:

Beam Ball FSI Benchmark Sketch



The ball (sphere), with diameter D = 30 cm, is closely fixed to the beam (cylindrical rod), with length L = 1 m and diameter d = 1 cm, which is fixed in the wall. The beam and ball material is steel. The beam-ball system is exposed to the airflow, which velocity is U=30 m/s. The airflow acts on the beam-ball system and the beam bends due to the aerodynamic force. Let’s determine the beam deformation (displacement at the free end of the beam where the ball is fixed). The other objective is finding the material stress distribution, particularly the maximal stress (to be able to compare results from 1D analysis to 3D simulation results). This task can be solved both analytically, and numerically using special CAE software.

The analytical solution can be done per-partes. First, the aerodynamics drag force, which is acting on the ball, can be evaluated from a physical formula (and double-checked with the simulation). Second, the resulting aerodynamic drag force is used as a point load at a 1D analysis of a cantilever beam, fixed in the wall. The correctness of the 1D analysis below can be also verified with the quick online cantilever beam calculator[1].



Within the numerical simulation, the solution is obtained using a combination of a standard 3D CFD simulation, FSI (Fluid-Structure Interaction), and standard 3D FEA (Finite Element Analysis) simulation. First, CFD simulation is performed. The resulting pressure field, which is acting on the ball surface, is converted with the FSI module into the pressure forces. The pressure forces are used as an input (load) for the FEA simulation, which evaluates complete deformation and stress volume fields.

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[1] https://calcresource.com/statics-cantilever-beam.html

Ball Aerodynamics - Drag Force Analytical solution

The drag force acting on any object can determined by a simple formula:

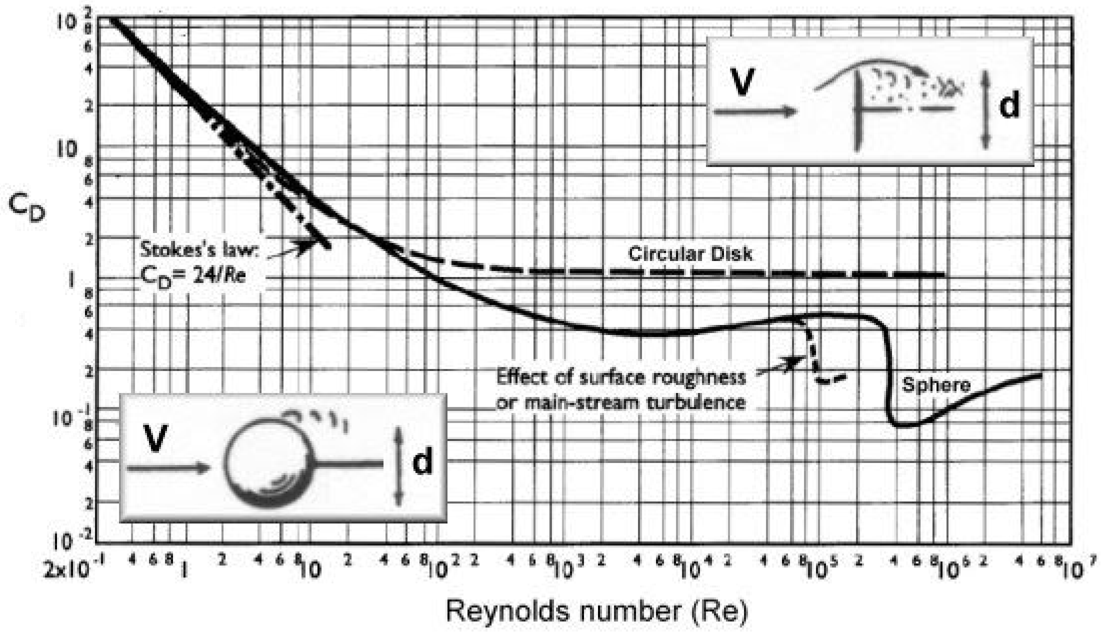
$$F = 0.5 \cdot C_d \cdot A_{ref} \cdot \rho \cdot U_\infty^2$$

where

- A_{ref} is a reference area. In this case it is an area of a circle with the same diameter as the analysed sphere
- ρ is density of fluid
- U_∞ is velocity of the mean flow
- C_d is drag coefficient

Drag coefficient of the ball

The drag coefficient, C_d, of the ball (sphere) is depending on the Reynolds Number[2].



$$Re = \frac{\rho \cdot U_\infty \cdot D}{\mu} = 600\ 000$$

Estimation of the ball drag coefficient is not straightforward. Considering the Reynolds number of 600000, according to NASA documentation[3], the drag coefficient can be estimated in a range from $C_d = 0.1 - 0.5$, which leads to evaluating the drag force acting on the ball to:

$$F = 0.5 \cdot C_d \cdot A_{ref} \cdot \rho \cdot U_\infty^2 \simeq 3.807 - 19.035\ [N]$$

The CFD simulation, described below, gives the drag coefficient $C_d = 0.215$, which leads to the drag force:

$$F = 0.5 \cdot C_d^{simulation} \cdot A_{ref} \cdot \rho \cdot U_\infty^2 \simeq 8.22462\ [N]$$

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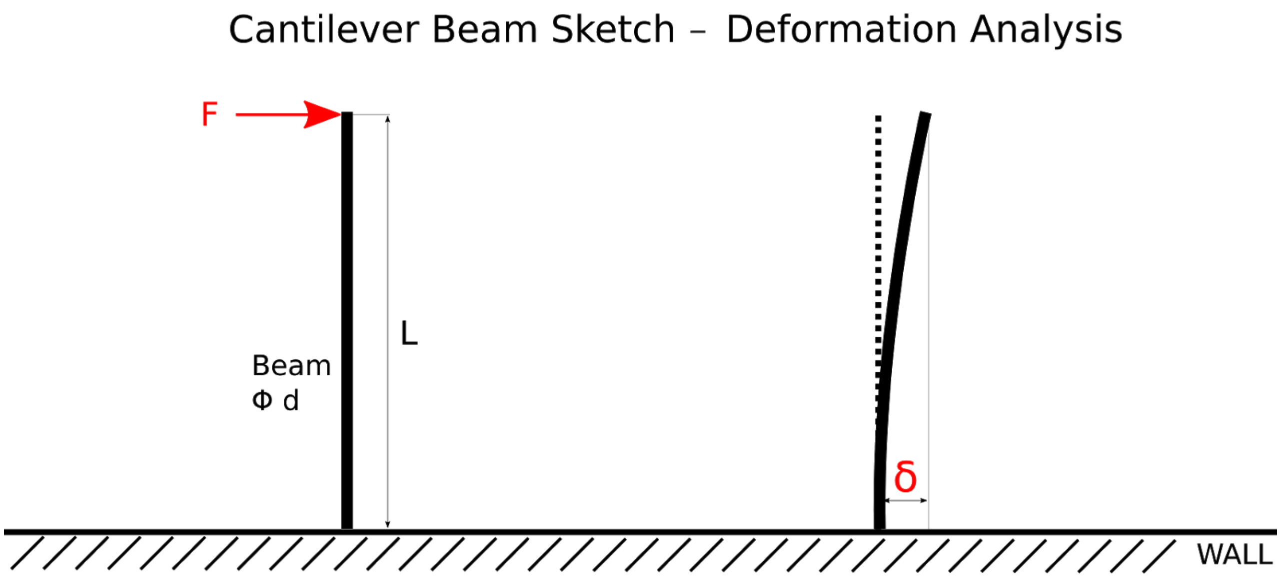
[2] Ball Cd image credit http://www.aerospaceweb.org/question/aerodynamics/q0231.shtml

[3] https://www.grc.nasa.gov/www/k-12/airplane/dragsphere.html

Beam Deformation and Stress - Analytical solution

Deformation Analysis

Let’s consider a simplified one-dimensional model of a cantilever beam, with one fixed end and one free end. Let the beam length is L and the beam is loaded with a single point load (force F) at its end.



We can obtain analytical solution of beam displacement, $\delta_{(x)}$, anywhere along the beam by applying *Mohr integral formula* [4]

$$\delta_{(x)} = \frac{1}{EI} \int\limits_{(L)} M_{o\,(x)} \cdot m_{o\,(x)}\, dx \quad [m]$$

Where

- $M_o(x)$ denotes *Bending Moment Function*, along the beam length (x - coordinate, x=0 at the free end and x=L at the wall), from load by the force F
- $m_o(x)$ means *Bending Moment*, along the beam length (x - coordinate, x=0 at the free end and x=L at the wall), from unit dimensionless force (“1”) applied at the very point, where we want to analyse displacement
- E is *Young Modulus of Elasticity* of the beam material
- I represents the Moment of Inertia of a beam cross-section. For circular beams it is

$$I = \frac{\pi \cdot d^4}{64}$$

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[4] E.g. https://en.wikipedia.org/wiki/Moment-area_theorem

In our particular case, the moment functions are

$$M_{o\,(x)} = F \cdot x$$

$$m_{o\,(x)} = "1" \cdot x$$

Which gives us integral

$$\delta_{(x)} = \frac{1}{EI}\int_0^L F \cdot x^2\, dx$$

After integration we obtain

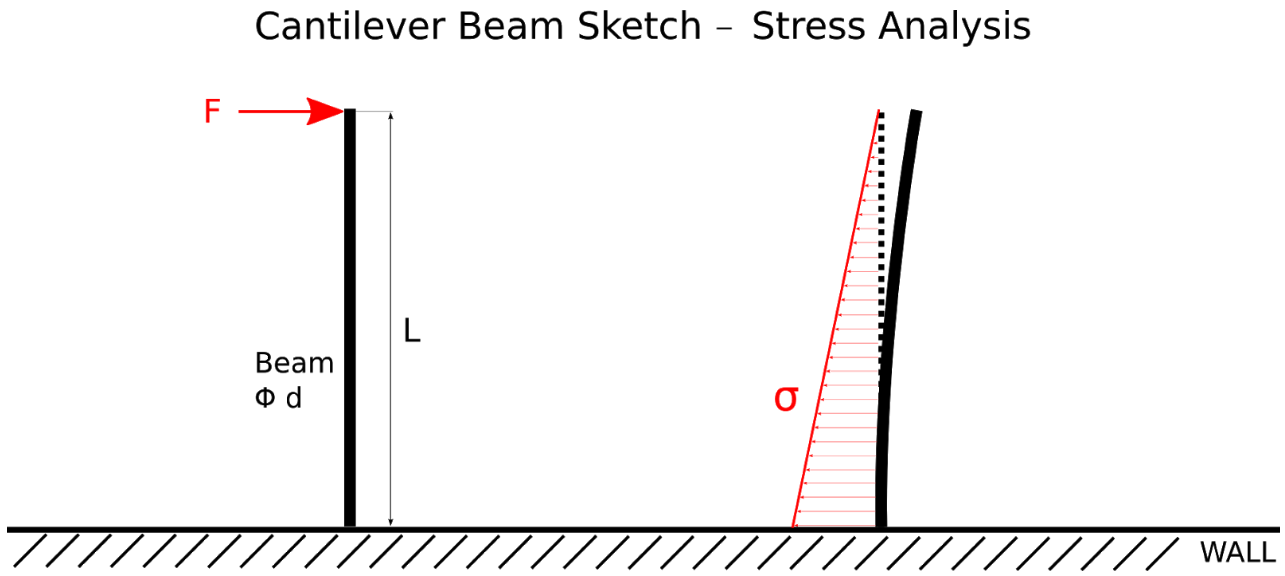
$$\delta = \frac{1}{EI}\left[F \cdot \frac{x^3}{3}\right]_0^L = \frac{1}{EI}\left[\frac{FL^3}{3}\right]$$

Which can be simplified into a form

$$\delta = 0.003234 \cdot F = 0.026598\ [m]$$

Stress Analysis

Let’s consider the same cantilever beam, with one fixed end and one free end. Let the beam length is *L* and the beam is loaded with a single point load (force F) at its end.



Now we derive the material stress $\sigma_{o(x)}$ in the beam

$$\sigma_{o(x)} = \frac{M_{o(x)}}{W_o} \quad [Pa]$$

where

- $\sigma_{o(x)}$ is the *Stress Function*, along the beam length (x - coordinate, x=0 at the free end and x=L at the wall), from load by the force F
- $M_o(x)$ denotes *Bending Moment Function*, along the beam length (x - coordinate, x=0 at the free end and x=L at the wall), from load by the force F
- W_o declares *Resistant Moment* of the neutral axis of bending, which is Moment of Inertia divided by the longest distance of thread from the neutral axis, in our case of circular beam it is

$$W_o = \frac{I}{y} = \frac{\frac{\pi \cdot d^4}{64}}{\frac{d}{2}} = \frac{\pi \cdot d^3}{32} = 9.8172 \cdot 10^{-8} m^3$$

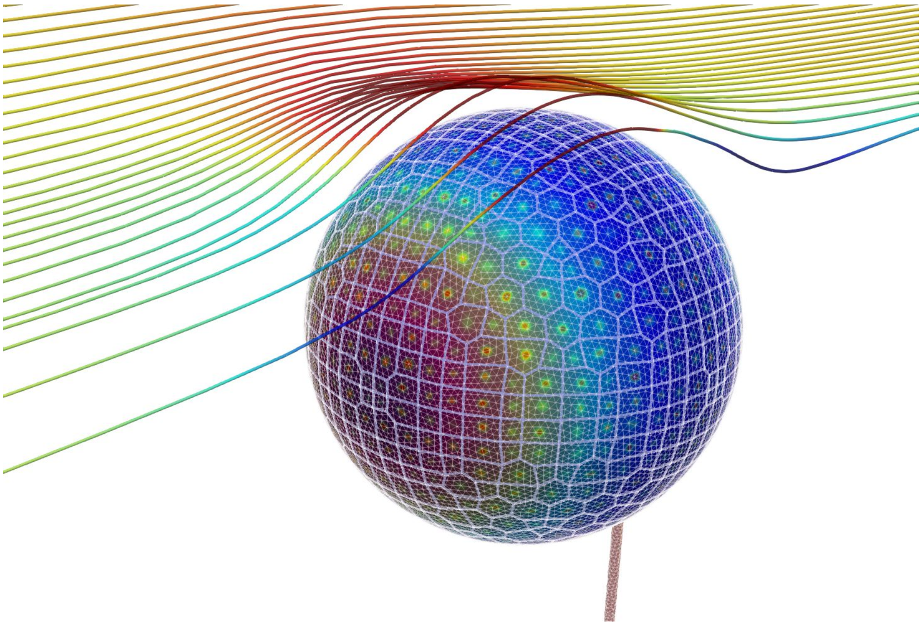
In the structural analysis, we are typically interested in the maximal value of the stress. That can be found by the derivation of stress or moment function to find maximal or minimal values

$$\frac{\partial \sigma_{o(x)}}{\partial x} = 0$$

In our model, we can simply assume, that the stress function is linear and the maximal stress will be at the wall ($x = L$), so

$$\sigma_{o_{max}} = \frac{M_{o(x=L)}}{W_o} = \frac{F \cdot L}{W_o}$$

$$\sigma_{o_{max}} = 1.0186 \cdot 10^7 \cdot F = 8.3776 \cdot 10^7 \quad [Pa]$$



Beam-Ball Full Case Simulation in TCAE

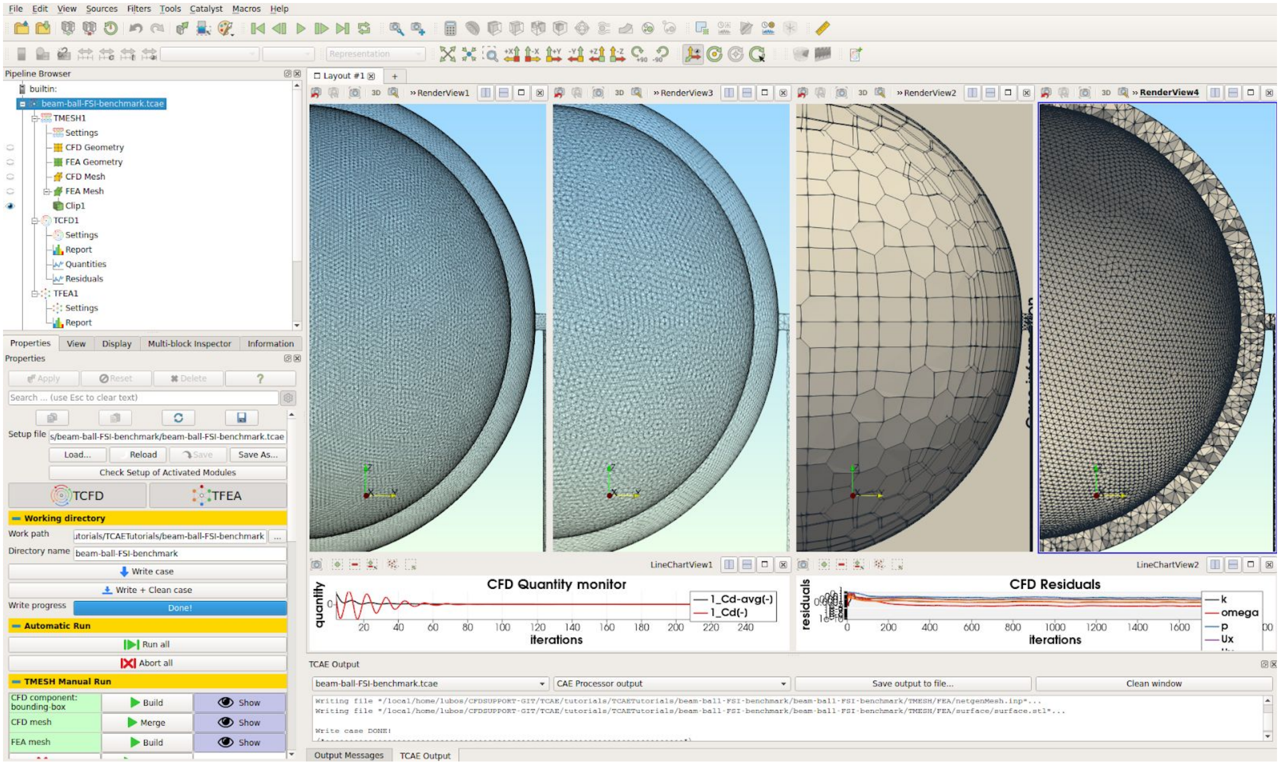
The beam-ball surface model is created in CAD and loaded in TCAE in the form of fine 3D STL surface files. All the simulation setup is done in the TCAE graphical interface.

The TCAE simulation workflow is completely automated. If the simulation setup is completed, the whole simulation process is run with a single click of the *Run all* button in GUI or s single command is executed in the command line.

TCAE consists of different software modules. The simulation workflow chronological order is following: **Start -> CFD mesh -> FEA mesh -> CFD simulation -> FSI -> FEA simulation -> Report**.

This particular case uses the following TCAE software modules: TMESH module creates meshes both for CFD and FEA and makes them ready for the simulation. TCFD module runs CFD simulation and writes down the CFD results report. TFEA module makes FSI conversion, runs FEA simulation, and writes down the FEA results report.

The aerodynamic effects acting on the beam itself are neglected. The aerodynamic simulation is performed only around the ball (sphere) to benefit from completely symmetrical mesh to receive as accurate results as possible. Note, the simulation results will be compared with an analytical solution, which also considers the point load at the end of the beam.



Setup file: s/beam-ball-FSI-benchmark/beam-ball-FSI-benchmark.tcae

Work path: utorials/TCAETutorials/beam-ball-FSI-benchmark

Directory name: beam-ball-FSI-benchmark

Write progress: Done!

CFD component: bounding-box

CFD mesh

FEA mesh

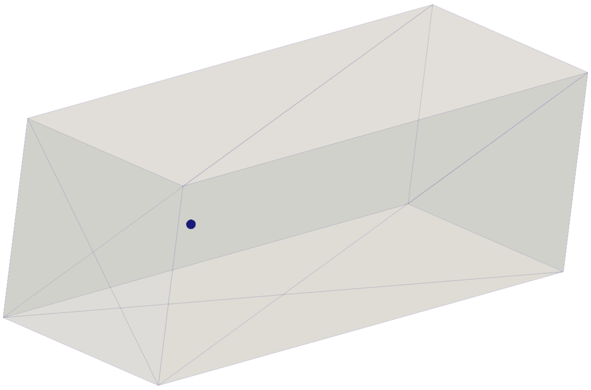
beam-ball-FSI-benchmark.tcae

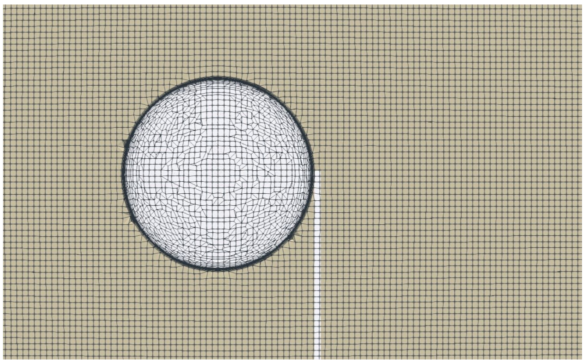
CAE Processor output

```
Writing file "/local/home/lubos/CFDSUPPORT-GIT/TCAE/tutorials/TCAETutorials/beam-ball-FSI-benchmark/beam-ball-FSI-benchmark/TMESH/FEA/netgenMesh.inp"...
Writing file "/local/home/lubos/CFDSUPPORT-GIT/TCAE/tutorials/TCAETutorials/beam-ball-FSI-benchmark/beam-ball-FSI-benchmark/TMESH/FEA/surface/surface.stl"...

Write case DONE!
```

CFD Mesh



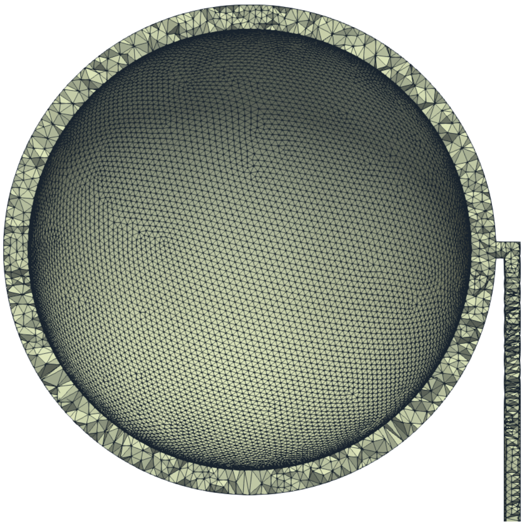


The computational mesh for CFD was created in an automated software module TMESH, using the *snappyHexMesh* open-source application. For each model component (in this case, just one component - Virtual Tunnel), a cartesian block mesh was created, as an initial background mesh, that is further refined along with the simulated object. The whole virtual tunnel is *20 m* long, *8 m* high, and *8 m* wide. Basic mesh cell size is a cube of 200mm edge. Three additional refinement regions are added. The mesh is gradually refined to the model wall. The mesh refinement levels can be easily changed, to obtain the coarser or finer mesh, to better handle the mesh size.Inflation layers can be easily handled. In this particular case, there are *5* inflation layers with an expansion ratio of *1.2*.
The final CFD mesh used for the simulation has in total 489,620 cells and consists mostly of hexahedrons.

The meshing application snappyHexMesh is not a compulsory meshing tool for TMESH at all. In case of need, any other external mesh can be loaded in TMESH directly in *MSH*, *CGNS*, or *OpenFOAM* format.

Mesh Elements	points	faces	internal faces	faces per cell	hexahedra
#	*535722*	*1512960*	*1487592*	*6.128328*	*458023*
Mesh Elements	prisms	pyramids	tet wedges	polyhedra	**cells**
#	*1056*	*0*	*0*	*30541*	***489620***

FEA Mesh



The computational mesh for CFD was created in an automated software module TMESH, using *NetGen* open-source application.

- Number of Points: *63135*
- Number of Elements: *189666*
- Number of Surface Elements: *124742*
- Shortest Edge: *0.000378 [mm]*
- Longest Edge: *0.010111 [mm]*
- Minimal Angle Between Edges: *6.97633 [deg]*
- Maximal Ang. Between Edges: *145.514 [deg]*
- Minimal Angle Between Faces: *4.94836 [deg]*
- Maximal Angle Between Faces: *169.359 [deg]*

CFD Simulation Case Setup

The CFD simulation is managed with TCAE software module TCFD. Complete CFD simulation setup and run is done in the TCFD GUI in ParaView. TCFD uses OpenFOAM open-source application.

- Simulation type: *Virtual tunnel*
- Time management: *steady-state*
- Physical model: *Incompressible*
- Number of components: *1 [-]*
- Wall roughness: none
- Inlet: *Velocity 30 [m3/s]*
- Outlet: *Static pressure 0 [m2/s2]*
- Turbulence: *RANS*
- Turbulence model: *k-omega SST*
- Wall treatment: *Wall functions*
- Turbulence intensity: 1%
- Speedlines: *1 [-]*
- Simulation points: *1 [-]*
- Fluid: *Air*
- Reference pressure: *1 [atm]*
- Dynamic viscosity: *1.8 × 10^{-5} [Pa · s]*
- Air density: *1.2 [kg/m^3]*
- CFD CPU Time: *2 core.hours*

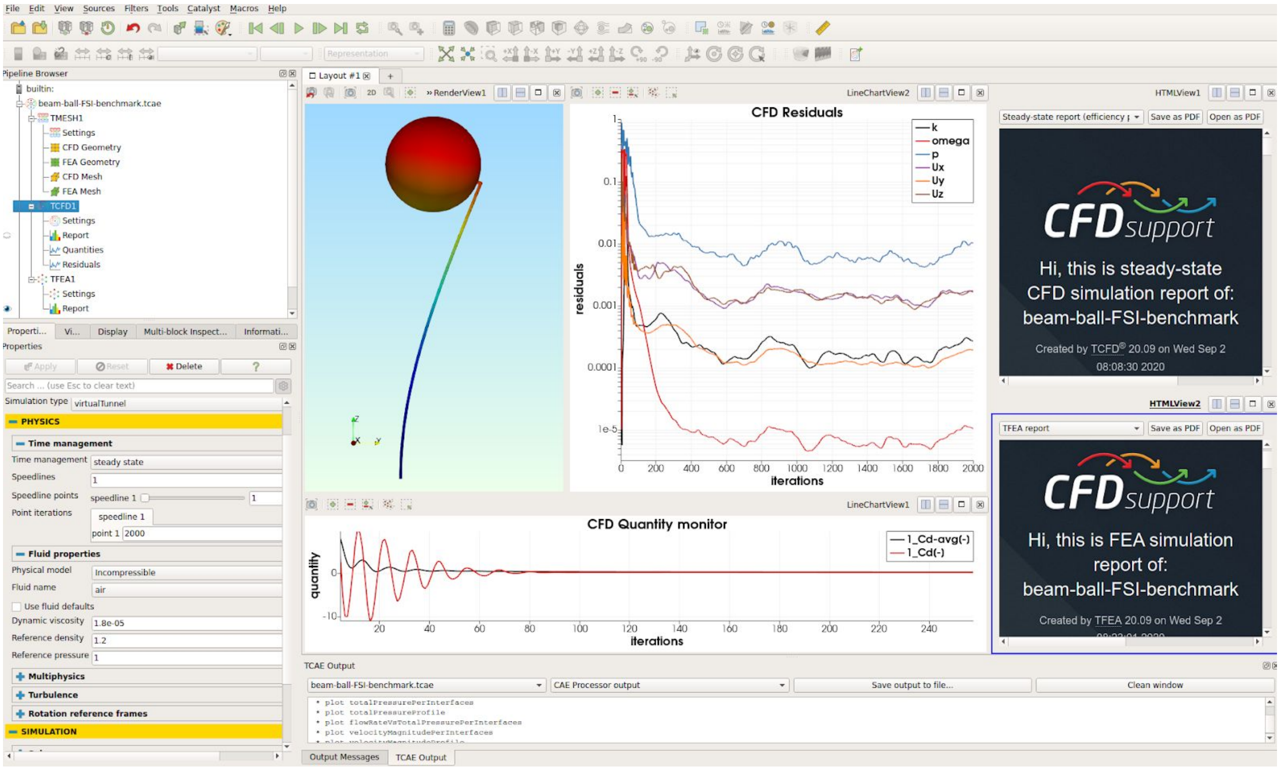
FEA Simulation Case Setup

The FEA simulation is managed with TCAE software module TFEA. Complete FEA simulation setup and run is done in the TFEA GUI in ParaView. TFEA uses Calculix open-source application..

- Beam material: *steel*
- Material density: *7800 kg/m^3*
- Material structure: *isotropic*
- Young modulus: *2.1E11 Pa*
- Poisson ratio: *0.3*
- Simulation type: *Virtual Tunnel*
- Finite element order: *second*
- FEA CPU Time: *0.1 core.hours*

Simulation run

The simulation was executed within the automated TCAE workflow in steady-state mode, for one flow velocity value of 30 m/s. TCFD is capable of writing the results down at any time during the simulation. The convergence of any quantity is monitored during the simulation.



Simulation type: virtualTunnel

PHYSICS

Time management

Time management: steady state

Speedlines: 1

Speedline points: speedline 1: 1

Point iterations: speedline 1: point 1: 2000

Fluid properties

Physical model: Incompressible

Fluid name: air

☐ Use fluid defaults

Dynamic viscosity: 1.8e-05

Reference density: 1.2

Reference pressure: 1

Multiphysics

Turbulence

Rotation reference frames

SIMULATION

CFD support

Hi, this is steady-state CFD simulation report of: beam-ball-FSI-benchmark

Created by TCFD® 20.09 on Wed Sep 2 08:08:30 2020

CFD support

Hi, this is FEA simulation report of: beam-ball-FSI-benchmark

Created by TFEA 20.09 on Wed Sep 2

```
* plot totalPressurePerInterfaces
* plot totalPressureProfile
* plot flowRateVsTotalPressurePerInterfaces
* plot velocityMagnitudePerInterfaces
```

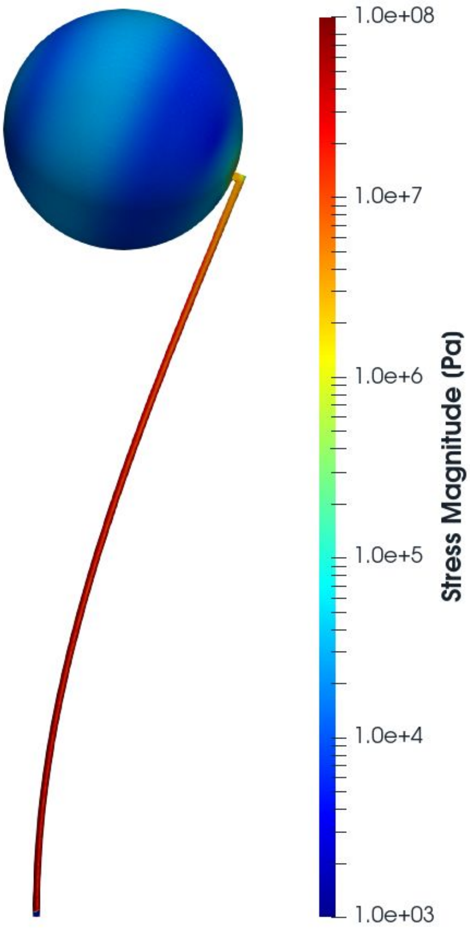
Results

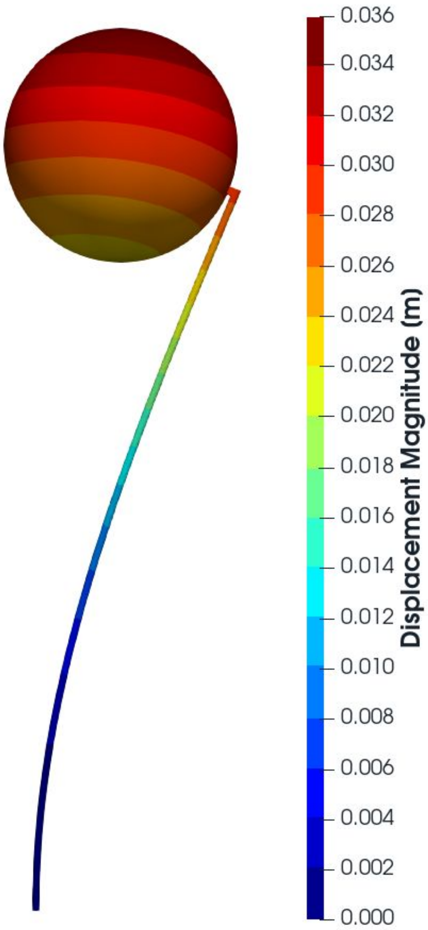
The table below shows the final comparison of the analytical solution with the simulation results.

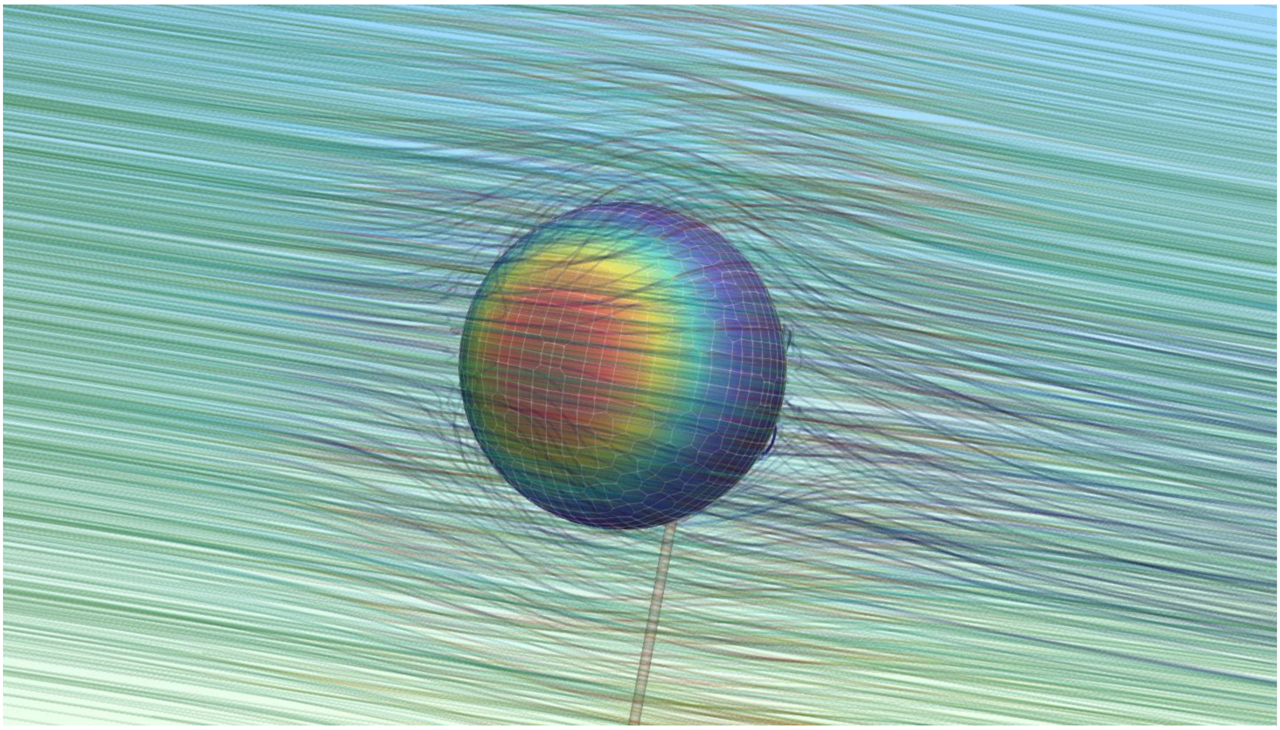
	Analytical solution	TCAE simulation
Max Deformation [mm]	26.60	27.75
Max Stress [MPa]	83.78	92.48

The compared results differ by about 5% in maximal deformation and 10% in maximal stress which meets well the original expectation. Despite the analytical 1d solution can be assumed as final, the simulation results can slightly differ according to the simulation settings. Especially, the CFD mesh and FEA mesh density may have certain effects on the simulation results.

The following figures show the deformed beam stress magnitude and displacement magnitude. The deformation on the figures is ten times larger for a better visual perception.







Conclusion

- The complex FSI analysis was performed successfully. It has been shown that the 3D TCAE simulation prediction gives a good agreement with the 1D analytical solution.
- This benchmark is completely open, transparent, and repeatable. All the case data, resources and results are freely available. Everyone is welcome to contribute with new research about this case.
- TCAE showed to be a very effective tool for CFD, FEA, and FSI engineering simulations.
- More information about TCAE can be found on CFD SUPPORT website: https://www.cfdsupport.com/tcae.html
- Questions will be answered via email *info@cfdsupport.com*

References

[1] TCFD Manual https://www.cfdsupport.com/download/TCFD-manual-v19.10.pdf

[2] TCFD Training https://www.cfdsupport.com/download/TCFD-Training-19.10.pdf

[3] Wikipedia https://en.wikipedia.org/wiki/Bending

[4] Ball Cd image credit http://www.aerospaceweb.org/question/aerodynamics/q0231.shtml

[5] Drag of a Sphere https://www.grc.nasa.gov/www/k-12/airplane/dragsphere.html

[6] E.g. https://en.wikipedia.org/wiki/Moment-area_theorem

[7] On-line beam calculator: https://calcresource.com/statics-cantilever-beam.html